MATH 115 Linear Algebra
Worksheet 9

December 31, 2020

1. For each of the matrices below find the eigenvalues and determine bases for the eigenspaces. For each eigenvalue, determine the multiplicity. Determine whether the matrices are diagonalizable or not. If diagonalizable, determine P and D where $D = P^{-1}AP$.

   (a) $A = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$.

   (b) $A = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix}$.

   (c) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

   (d) $A = \begin{bmatrix} 2 & 2 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$.

   (e) $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -3 & -1 \\ -1 & 1 & -1 \end{bmatrix}$.

   (f) $A = \begin{bmatrix} 15 & -32 & 12 \\ 8 & -17 & 6 \\ 0 & 0 & -1 \end{bmatrix}$.

   (g) $A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{bmatrix}$.

   (h) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 2 & -2 & -1 \end{bmatrix}$.

   (i) $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

   (j) $A = \begin{bmatrix} -1 & 1 & 0 \\ -2 & -3 & 1 \\ 1 & 1 & -2 \end{bmatrix}$.

   (k) $A = \begin{bmatrix} -2 & 3 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

Answers

1. (a) $\lambda_1 = 2$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 2$.

   (b) $\lambda_1 = -1$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 2$.

   (c) $\lambda_1 = 1$, $\lambda_2 = -1$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 1$, $E_{\lambda_2} = \text{span}\left\{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_2) = 2$.

   (d) $\lambda_1 = 2$, $\lambda_2 = 0$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ 2/3 \\ 4/3 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 1$, $E_{\lambda_2} = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right\}$, $m_a(\lambda_2) = 2$.

   (e) $\lambda_1 = -2$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (f) $\lambda_1 = -1$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -4/3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (g) $\lambda_1 = 4$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (h) $\lambda_1 = 1$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (i) $\lambda_1 = 4$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (j) $\lambda_1 = -2$, $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 3$.

   (k) $\lambda_1 = -5$, $\lambda_2 = 1$. $E_{\lambda_1} = \text{span}\left\{\begin{bmatrix} 1 \\ -1 \\ -5/36 \\ 1/6 \end{bmatrix}\right\}$, $m_a(\lambda_1) = 1$, $E_{\lambda_2} = \text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right\}$, $m_a(\lambda_2) = 3$.