

1. Find derivatives of the following functions. You may need to use chain rule.

(a)  $f(x) = (x^2 + 1)^2$

(b)  $f(x) = (x^2 + 1)^{100}$

(c)  $f(x) = \sin 4x \cos 5x$

(d)  $f(x) = \sqrt[3]{1 + \sec x}$

(e)  $f(x) = \frac{1}{(x^2 - x)^4}$

(f)  $f(x) = \frac{\sin \pi x}{\cos \pi x + \sin \pi x}$

(g)  $f(x) = \frac{x}{\sqrt{1 + x^2}}$

(h)  $f(x) = x \sin \sqrt[3]{x}$

(i)  $f(x) = (x^2 + 1)(\sqrt[4]{x^3 + 5})$

(j)  $f(x) = (x^4 - 1)^3(x^5 + 1)^6$

(k)  $f(x) = \frac{x}{\sqrt{3 - 2x}}$

(l)  $f(x) = \sin(\sin(\sin(x^2)))$

(m)  $f(x) = \sqrt[3]{x + \sqrt{x}}$

2. The derivatives of the exponential and logarithmic functions are as follows:

$$(\ln x)' = \frac{1}{x} \quad (e^x)' = e^x$$

Find derivatives of the following functions.

(a)  $e^{3x}$

(b)  $\ln x^3$

(c)  $(x^4 + 2x)2^x$

(d)  $\log_2 x$

(e)  $3^x$

(f)  $\ln \sqrt{x^2 + 6}$

(g)  $e^{(\sin x + x^4)}$

(h)  $\log(10x\sqrt{x-3})$

(i)  $\frac{4}{e^{-x} + e^x}$

(j)  $\log \frac{\sqrt[3]{x+2}}{\cos 5x}$

(k)  $\ln \frac{x^2 \sin^3 x}{\sqrt{x^2 + 1}}$

(l)  $\ln \sqrt{\frac{1}{x^3 + 5}}$

3. Find derivatives. (Hint: It may be useful to take logarithm firstly, (for example,  $5^{\sqrt{x}} = e^{\ln 5^{\sqrt{x}}} = e^{\sqrt{x} \cdot \ln 5}$ ) and then derivate.)

(a)  $5^{\sqrt{x}}$

(c)  $(\ln x)^{\cos x}$

(b)  $x^x$

(d)  $x^{\tan x}$

4. • Find  $y'$  by implicit differentiation.

• Solve the equation explicitly for  $y$  and differentiate to get  $y'$  in terms of  $x$ .

• Check that your solutions are consistent.

(a)  $9x^2 - y^2 = 1$

(b)  $1/x + 1/y = 1$

5. Find  $\frac{dy}{dx}$  by implicit differentiation.

(a)  $x^3 + y^3 = 1$

(b)  $x^4(x + y) = y^2(3x - y)$

(c)  $4 \cos x \sin y = 1$

(d)  $\tan(x/y) = x + y$

(e)  $\sqrt{xy} = 1 + x^2y$

(f)  $y \cos x = 1 + \sin(xy)$