

1. Find the absolute maximum and minimum values of the following functions:
  - (a)  $f(x) = x^5 - x^3 + 2, \quad -1 \leq x \leq 1$
  - (b)  $f(x) = x^4 - 3x^3 + 3x^2 - x, \quad 0 \leq x \leq 2$
  - (c)  $f(x) = \frac{x}{x^2 - x + 1}, \quad 0 \leq x \leq 3.$
  - (d)  $f(x) = x\sqrt{x - x^2}$
  - (e)  $f(x) = x - 2\cos(x), \quad -2 \leq x \leq 0.$
2. For the following functions, (i) find the intervals on which  $f$  is increasing and decreasing, (ii) find the local maximum and minimum values of  $f$ .
  - (a)  $f(x) = 2x^3 + 3x^2 - 36x$
  - (b)  $f(x) = x^4 + 2x^2 + 3$
  - (c)  $f(x) = \frac{x}{x^2 + 1}$
  - (d)  $\sin(x) + \cos(x), \quad 0 \leq x \leq 2\pi$
3. Find two numbers whose difference is 100 and whose product is a minimum.
4. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?
5. What is the minimum vertical distance between the parabolas  $y = x^2 + 1$  and  $y = x - x^2$ ?
6. Find the dimensions of a rectangle with area  $1000 \text{ m}^2$  whose perimeter is as small as possible.