MATH 115 Linear Algebra
Worksheet 7

December 31, 2020

NOTE: In the problems below, $\mathbb{R}^n$ should be considered with their standard inner product.

1. Find the unit vectors in the direction of the following vectors.

   (a) $(1, 2)$ (b) $(1, -3, 7)$ (c) $(1, 2, 5)$.

2. Compute the following inner products:

   (a) $\langle(2, -1), (5, 1)\rangle$.
   (b) $\langle(0, 4), (-8, -1)\rangle$.
   (c) $\langle(-1, 4, 2), (4, 6, 1)\rangle$.

3. Find the value(s) of a that make the given vectors orthogonal:

   (a) $\vec{u} = (a, 1), \vec{v} = (2, -3)$.
   (b) $\vec{u} = (a, 2a, 4), \vec{v} = (-1, 4, 2)$.
   (c) $\vec{u} = (3a, 0, 4), \vec{v} = (a, 0, a)$.
   (d) $\vec{u} = (-6, a, 2), \vec{v} = (a, a^2, a)$.

4. Are the given pairs of vectors orthogonal?

   (a) $\vec{u} = (12, 3, -5)$, $\vec{v} = (2, -3, 3)$.
   (b) $\vec{u} = (3, 2, -5, 0)$, $\vec{v} = (-4, 1, -2, 6)$.
   (c) $\vec{u} = (-3, 7, 4, 0)$, $\vec{v} = (1, -8, 15, -7)$.

5. If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set in V and if c_1, c_2, and c_3 are scalars, then show that $\{c_1\vec{v}_1, c_2\vec{v}_2, c_3\vec{v}_3\}$ is an orthogonal set.

6. Show that an orthogonal set of nonzero vectors is linearly independent.

7. Determine which of the sets below are orthogonal. Is any of them an orthonormal set?

   (a) $\{(-1, 4, -3), (5, 2, 1), (3, -4, -7)\}$.
   (b) $\{(0, -1, 0, 0), (0, 0, 0, 1), (0, 0, -1, 0)\}$.
   (c) $\{(1, -2, 1), (0, 1, 2), (-5, -2, 1)\}$.
   (d) $\{(2, -7, -1), (-6, -3, 9), (3, 1, -1)\}$.
   (e) $\{(3, -2, 1, 3), (-1, 3, -3, 4), (3, 8, 7, 0)\}$.

Answers

1. (a) $\frac{1}{\sqrt{5}}(1, 2)$ (b) $\frac{1}{\sqrt{59}}(1, -3, 7)$ (c) $\frac{1}{\sqrt{30}}(1, 2, 5)$.
2. (a) 9 (b) -4 (c) 22.
3. (a) $3/2$ (b) $-8/7$ (c) $-4/3, 0$ (d) $-2, 0, 2$.
4. (a) Yes. (b) Yes. (c) No.
5.
6.
7. (a) Not orthogonal, so not orthonormal (b) Orthogonal and orthonormal (c) Orthogonal, but not orthonormal (d) Not orthogonal, so not orthonormal (e) Orthogonal, but not orthonormal.