Differential Equations **Worksheet 7** Name:

1. The general solution of the homogeneous equation

$$4x^2y'' + 9xy' + y = 0$$

can be written as

$$y_c = ax^{-1} + bx^{-1/4},$$

where a, b are arbitrary constants and

$$y_p = 1 + 2x$$

is a particular solution of the nonhomogeneous equation

$$4x^2y'' + 9xy' + y = 20x + 1.$$

By superposition, the general solution of the equation $4x^2y'' + 9xy' + y = 20x + 1$ is

$$y = \dots\dots\dots$$

Find the solution satisfying the initial conditions $y(1) = 3$, $y'(1) = 8$.

$$y = \dots\dots\dots$$

The Wronskian W of the fundamental set of solutions $y_1(x) = x^{-1}$ and $y_2(x) = x^{-1/4}$ for the homogeneous equation is

$$W = \dots\dots\dots$$

2. The general solution of the homogeneous differential equation

$$y'' + 4y' = 0$$

can be written as

$$y_c = c_1 \cos(2x) + c_2 \sin(2x),$$

where c_1, c_2 are arbitrary constants and

$$y_p = e^{-3x}$$

is a particular solution of the nonhomogeneous equation

$$y'' + 4y' = 13e^{-3x}.$$

By superposition, the general solution of the equation $y'' + 4y' = 13e^{-3x}$ is

$$y = \dots\dots\dots$$

Find the solution satisfying the initial conditions $y(0) = -1$, $y'(0) = 0$.

$$y = \dots\dots\dots$$

The Wronskian W of the fundamental set of solutions $y_1(x) = \cos(2x)$ and $y_2(x) = \sin(2x)$ for the homogeneous equation is

$$W = \dots\dots\dots$$

3. $y_1 = e^{3x}\cos(4x)$ and $y_2 = e^{3x}\sin(4x)$ are solutions to the differential equation $y'' - 6y' + 25y = 0$ on the interval $(-\infty, \infty)$. Find the Wronskian of y_1, y_2. Do the functions y_1, y_2 form a fundamental set on $(-\infty, \infty)$?

4. $y_1 = x^{-2}$, $y_2 = x^{-3}$ and $y_3 = 2$ are solutions to the differential equation $x^2y''' + 8xy'' + 12y' = 0$ on the interval $(0, \infty)$. Find the Wronskian of y_1, y_2, y_3. Do the functions y_1, y_2, y_3 form a fundamental set on $(0, \infty)$?

5. Given a second order linear homogeneous differential equation

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$$

we know that a fundamental set for this ODE consists of a pair linearly independent solutions y_1, y_2. But there are times when only one function, call it y_1, is available and we would like to find a second linearly independent solution. We can find y_2 using the method of reduction of order.

First, under the necessary assumption the $a_2(x) \neq 0$ we rewrite the equation as

$$y'' + P(x)y' + Q(x)y = 0, \qquad P(x) = \frac{a_1(x)}{a_2(x)}, \qquad Q(x) = \frac{a_0(x)}{a_2(x)},$$

Then the method of reduction of order gives a second linearly independent solution as

$$y_2(x) = u(x)y_1(x) = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx.$$

Given the problem

$$25y'' - 20y' + 4y = 0$$

and a solution $y_1 = e^{(2x/5)}$. Applying the reduction of order method to this problem we obtain the following

$$y_1^2(x) = \ldots\ldots$$
$$P(x) = \ldots$$
$$e^{-\int P(x)dx} = \ldots\ldots$$

Finally, we arrive at

$$y_2(x) = u(x)y_1(x) = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx = \ldots\ldots$$

So the general solution to $25y'' - 20y' + 4y = 0$ can be written as

$$y = c_1y_1 + c_2y_2 = \ldots\ldots$$

6. Given the problem

$$x^2y'' + 5xy' - 12y = 0$$

and a solution $y_1 = x^2$. Applying the reduction of order method to this problem we obtain the following

$$y_1^2(x) = \ldots\ldots$$
$$P(x) = \ldots$$
$$e^{-\int P(x)dx} = \ldots\ldots$$

Finally, we arrive at

$$y_2(x) = u(x)y_1(x) = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx = \ldots\ldots$$

So the general solution to $x^2y'' + 5xy' - 12y = 0$ can be written as

$$y = c_1y_1 + c_2y_2 = \ldots\ldots$$

7. Given
$$y'' - 4y = 2$$
and a solution $y_1 = e^{-2x}$. Apply the reduction of order method to this problem to obtain general solution.

8. Given
$$y'' + y' = 1$$
and a solution $y_1 = 1$. Apply the reduction of order method to this problem to obtain general solution.

9. Given
$$y'' - 3y' + 2y = 5e^{3x}$$
and a solution $y_1 = e^{x}$. Apply the reduction of order method to this problem to obtain general solution.

10. Given
$$y'' - 4y' + 3y = x$$
and a solution $y_1 = e^{x}$. Apply the reduction of order method to this problem to obtain general solution.