Mathematics I

Worksheet 6

Name:

1. Find derivatives of the following functions. You may need to use chain rule.

- (a) $f(x) = (x^2 + 1)^2$ (b) $f(x) = (x^2 + 1)^{100}$ (c) $f(x) = \sin 4x \cos 5x$ (d) $f(x) = \sqrt[3]{1 + \sec x}$ (e) $f(x) = \frac{1}{(x^2 - x)^4}$ (f) $f(x) = \frac{\sin \pi x}{\cos \pi x + \sin \pi x}$ (g) $f(x) = \frac{x}{\sqrt{1 + x^2}}$ (h) $f(x) = x \sin \sqrt[3]{x}$ (i) $f(x) = (x^2 + 1)(\sqrt[4]{x^3 + 5})$ (j) $f(x) = (x^4 - 1)^3(x^5 + 1)^6$ (k) $f(x) = \frac{x}{\sqrt{3 - 2x}}$ (l) $f(x) = \sin(\sin(\sin(x^2)))$ (m) $f(x) = \sqrt[3]{x + \sqrt{x}}$
- 2. The derivatives of the exponential and logarithmic functions are as follows:

$$(\ln x)' = \frac{1}{x}$$
 $(e^x)' = e^x$

Find derivatives of the following functions.

(a) e^{3x} (b) $\ln x^3$ (c) $(x^4 + 2x)2^x$ (d) $\log_2 x$ (e) 3^x (f) $\ln \sqrt{x^2 + 6}$ (g) $e^{(\sin x + x^4)}$ (h) $\log(10x\sqrt{x - 3})$ (i) $\frac{4}{e^{-x} + e^x}$ (j) $\log \frac{\sqrt[3]{x + 2}}{\cos 5x}$ (k) $\ln \frac{x^2 \sin^3 x}{\sqrt{x^2 + 1}}$ (l) $\ln \sqrt{\frac{1}{x^3 + 5}}$

- 3. Find derivatives. (Hint: It may be useful to take logarithm firstly, (for example, $5^{\sqrt{x}} = e^{\ln 5^{\sqrt{x}}} = e^{\sqrt{x} \cdot \ln 5}$) and then derivate.)
 - (a) $5^{\sqrt{x}}$ (c) $(\ln x)^{\cos x}$
 - (b) x^x (d) $x^{\tan x}$
- 4. Find y' by implicit differentiation.
 - Solve the equation explicitly for y and differentiate to get y' in terms of x.
 - Check that your solutions are consistent.
 - (a) $9x^2 y^2 = 1$
 - (b) 1/x + 1/y = 1
- 5. Find $\frac{dy}{dx}$ by implicit differentiation.
 - (a) $x^3 + y^3 = 1$
 - (b) $x^4(x+y) = y^2(3x-y)$
 - (c) $4\cos x \sin y = 1$
 - (d) $\tan(x/y) = x + y$
 - (e) $\sqrt{xy} = 1 + x^2 y$
 - (f) $y \cos x = 1 + \sin(xy)$