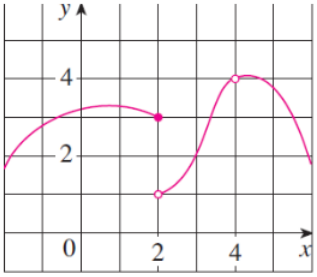
Mathematics I **Worksheet 4** Name:

1. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

   (a) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 2} f(x)$ (e) $\lim_{x\to 4} f(x)$

   (b) $\lim_{x\to 2^+} f(x)$ (d) $f(2)$ (f) $f(4)$



2. Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ does not exist.

   (a) $f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$

   (b) $f(x) = \begin{cases} 1+\sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \le x \le \pi \\ \sin x & \text{if } x > \pi \end{cases}$

3. Evaluate the limit, if it exists.

   (a) $\lim_{x\to 5} \frac{x^2-5x+6}{x-5}$ (c) $\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

   (b) $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$ (d) $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$

(e) $\lim_{x\to 0}(\frac{1}{t}-\frac{1}{t^2+1})$

(f) $\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

4. Use the Squeeze Theorem to show that

(a) $\lim_{x\to 0}\sqrt{x^3+x^2}\sin\frac{\pi}{x}=0$

(b) $\lim_{x\to 0}x^4\cos\frac{2}{x}=0$

5. Find the limit, if it exists. If the limit does not exist, explain why.

(a) $\lim_{x\to\frac{1}{2}}\frac{2x-1}{|2x^3-x^2|}$

(b) $\lim_{x\to -2}\frac{2-|x|}{2+x}$

(c) $\lim_{x\to 0}(\frac{1}{x}-\frac{1}{|x|})$ 0

6. Let $f(x)=\begin{cases}x^2+1 & \text{if } x<1\\ (x-2)^2 & \text{if } x\geq 1\end{cases}$

(a) Find $\lim_{x\to 1^-}f(x)$ and $\lim_{x\to 1^+}f(x)$

(b) Does $\lim_{x\to 1}f(x)$ exist?

(c) Sketch the graph of f(x)

7. Find the limit or show that it does not exist.

(a) $\lim_{x\to\infty}\frac{1-x^2}{x^3-x+1}$

(b) $\lim_{x\to\infty}\frac{x-x\sqrt{x}}{2x^{\frac{3}{2}}+3x-5}$

(c) $\lim_{x\to\infty}\frac{x^2}{\sqrt{x^4+1}}$

(d) $\lim_{x\to\infty}(\sqrt{9x^2+x}-3x)$

8. Find the limit.

(a) $\lim_{x\to 0}\frac{\sin 3x\sin 5x}{x^2}$

(b) $\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$