MATH 115 Linear Algebra
Worksheet 4

November 23, 2020

1. Find the inverses of the matrices using the formula given in the lecture (which is derived from Cramer's rule).

$$\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}, \begin{pmatrix} 3 & 1 & 1 \\ -2 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

2. Use Cramer's rule to solve the following systems.

   (a)
   $$\begin{aligned} 3x_1 - 2x_2 + x_3 &= 4 \\ x_1 + x_2 - x_3 &= 2 \\ x_1 + x_3 &= 1 \end{aligned}$$

   (b)
   $$\begin{aligned} x_1 - 2x_2 + 3x_3 - x_4 &= 1 \\ 2x_1 + x_3 &= 2 \\ x_1 + x_2 - x_4 &= 0 \\ x_2 - 2x_3 + x_4 &= 3 \end{aligned}$$

3. Find the area of the triangle with the vertices given below.

   (a) $(1,2), (2,2), (2,4)$.

   (b) $(1,1), (1,1), (0,2)$.

4. Determine whether the following points are collinear.

   (a) $(1,3), (4,7), (2,13)$.

   (b) $(2,5), (0,1), (3,9)$.

5. Find an equation of the line passing through the following points.

   (a) $(2,3), (2,4)$.

   (b) $(4,7), (2,4)$.

6. Find the volume of the tetrahedron with the given vertices.

   (a) $(1,1,1), (0,0,0), (2,1,1), (1,1,2)$.

   (b) $(3,3,3), (3,1,3), (3,1,3), (2,3,2)$.

7. Determine whether the following points are coplanar.

   (a) $(1,2,7), (3,6,6), (4,4,2), (3,3,4)$.

   (b) $(0,0,1), (0,1,0), (1,1,0), (2,1,2)$.

8. Find an equation of the plane passing through the following points.

   (a) $(0,1,0), (1,1,0), (2,1,2)$.

   (b) $(0,0,0), (1,1,0), (0,1,1)$.

9. Let $\vec{v} = (4,1)$ and $\vec{w} = (-2,2)$.
   Draw $\vec{v}, \vec{w}, \vec{v}+\vec{w}, \vec{v}-\vec{w}$ and $\vec{w}-\vec{v}$ in the plane.

10. If $\vec{v} - \vec{w} = (4,1)$ and $\vec{v} + \vec{w} = (-1,5)$ find $\vec{v}$, $\vec{w}$.

11. Write $\vec{w} = (1,9)$ as a linear combination of $\vec{u} = (1,2)$ and $\vec{v} = (3,-1)$.

12. Write $\vec{v} = (2,-3,4)$ as a linear combination of $\vec{v}_1 = (1,1,1)$, $\vec{v}_2 = (1,1,0)$ and $\vec{v}_3 = (1,0,0)$.

Answers

1. $\begin{pmatrix} 4/9 & 1/9 \\ -1/9 & 2/9 \end{pmatrix}, \begin{pmatrix} 3/8 & -1/8 & -3/8 \\ -3/8 & 1/8 & 11/8 \\ 1/4 & 1/4 & -1/4 \end{pmatrix}, \begin{pmatrix} 1 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}.$

2. (a) $(\frac{3}{2}, 0, -\frac{1}{2})$, (b) $(\frac{11}{3}, -\frac{17}{3}, -\frac{16}{3}, -2)$.

3. (a) 1, (b) 0.
4. (a) not collinear, (b) not collinear.
5. (a) $x = 2$, (b) $3x - 2y + 2 = 0$.
6. (a) $\frac{1}{6}$, (b) 0.
7. (a) not coplanar, (b) not coplanar.
8. (a) $y = 1$, (b) $x - y + z = 0$.
9.
10. $\vec{v} = (\frac{3}{2}, 3), \vec{w} = (\frac{-5}{2}, 2)$.
11. $4\vec{u} - \vec{v} = \vec{w}$.
12. $4\vec{v_1} - 7\vec{v_2} + 5\vec{v_3} = \vec{v}$.