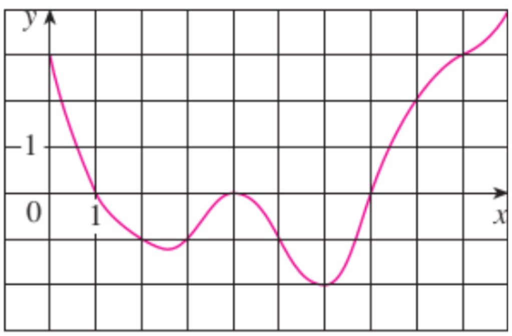
Mathematics II **Worksheet 3** Name:

1. Evaluate the Riemann sum for $f(x) = 1+\frac{2}{3}x^2$, $1 \leq x \leq 5$ with six subintervals, taking the samplepoints to be left endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

2. The graph of a function f is given. Estimate $\int_0^{10} f(x)dx$ using five subintervals with (a) right endpoints, (b) left endpoints.



3. A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_{10}^{30} f(x)dx$.

x	10	14	18	22	26	30
$f(x)$	-12	-6	-2	1	3	8

4. Express the limit as a definite integral on the given interval.

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i} \Delta x, \quad [\pi, 2\pi].$

   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} [5(x_i)^3 - 4(x_i)]\Delta x, \quad [2, 7].$

5. Use the formal definition of definite integral to calculate:

   (a) $\int_{-2}^{0} (x^2 + x)dx.$

   (b) $\int_{1}^{4} (x^2 - 4x + 2)dx.$

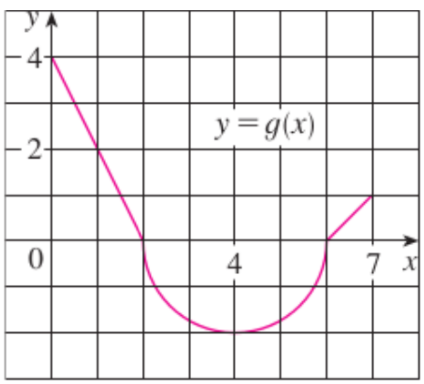
6. Prove that $\displaystyle\int_a^b x dx = \frac{b^2 - a^2}{2}$.

7. Express the integral as a limit of Riemann sums. Do not evaluate the limit.

(a) $\displaystyle\int_2^6 \frac{x}{1+x^5} dx$.

(b) $\displaystyle\int_0^{2\pi} x^2 \sin x dx$.

8. The graph of $g(x)$ consists of two straight lines and a semicircle. Use it to evaluate each integral (geometrically).



(a) $\displaystyle\int_0^2 g(x)\, dx$ (b) $\displaystyle\int_2^6 g(x)\, dx$ (c) $\displaystyle\int_0^7 g(x)\, dx$

9. Evaluate the integral by interpreting it in terms of areas (geometrically).

(a) $\displaystyle\int_{-3}^0 (1+\sqrt{9-x^2})\, dx$ (b) $\displaystyle\int_{-1}^2 |x|\, dx$ (c) $\displaystyle\int_{-5}^5 (x-\sqrt{25-x^2})\, dx$

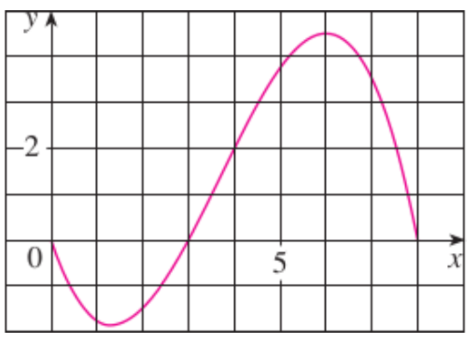
10. Evaluate $\displaystyle\int_\pi^\pi \sin^2 x \cos^4 x dx$.

11. Write the following expression as a single integral:

$$\int_{-2}^2 f(x)\, dx + \int_2^5 f(x)\, dx - \int_{-2}^{-1} f(x)\, dx$$

12. Find $\int_0^5 f(x)\,dx$, where $f(x) = \begin{cases} 3 & \text{if } x < 3 \\ x & \text{if } x \geq 3 \end{cases}$.

13. For the function f whose graph is shown, list the following quantities in increasing order, from smallest to largest.



(a) $\int_0^8 f(x)\,dx$ (b) $\int_0^3 f(x)\,dx$ (c) $\int_3^8 f(x)\,dx$ (d) $\int_4^8 f(x)\,dx$