(1) An experiment consists of tossing a coin three times. What is the sample space of this experiment? Which event corresponds to the experiment resulting in more heads than tails?

(2) Let $S = \{1, 2, 3, 4, 5, 6, 7\}$, $E = \{1, 3, 5, 7\}$, $F = \{7, 4, 6\}$, $G = \{1, 4\}$. Find

A) EF B) EG^c C) $E^c(F \cup G)$ D) $E \cup FG$ E) $EF^c \cup D$ F) $EG \cup FG$

(3) In the city of Lubbock, applications for zoning changes go through a two-step process: a review by the planning commission and a final decision by the city council. At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change. At step 2 the city council reviews the planning commission's recommendation and then votes to approve or to disapprove the zoning change. Suppose the developer of an apartment complex submits an application for a zoning change. Consider the application process as an experiment.

a. How many sample points are there for this experiment? List the sample points.
b. Construct a tree diagram for the experiment.

(4) Simple random sampling uses a sample of size n from a population of size N to obtain data that can be used to make inferences about the characteristics of a population. Suppose that, from a population of 50 bank accounts, we want to take a random sample of four accounts in order to learn about the population. How many different random samples of four accounts are possible?

(5) Consider two urns. The first contains two white and seven black balls, and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toast was head given that a white ball was selected?

(6) In answering a question on a multiple-choice test a student either knows the answer or guesses. Let p be the probability that she knows the answer and $1 - p$ the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{m}$, where m is the number of the multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that she answered it correctly?

(7) Consider a manufacturing firm that receives shipments of parts from two different suppliers. Currently 65% of parts purchased from Supplier 1 and 35% of parts purchased from Supplier 2. The quality rating of two suppliers:

TABLE 1.

	Percentage of Good Parts	Percentage of Bad Parts
Supplier 1	98	2
Supplier 2	95	5

Suppose now that the parts from the two suppliers are used in the firm's manufacturing process and that a machine breaks down because it attempts to process a bad part.

Given the information that the part is bad, what is the probability that it come from Supplier 1 and what is the probability that it come from Supplier 2?

(8) A box contains three marbles - one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box, then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat for the case in which the second marble is drawn without first replacing the first marble.

TABLE 2.

A_i	$P(A_i)$	$P(B\|A_i)$	$P(A_i \cap B)$	$P(A_i\|B)$
A_1				
A_2				

(9) A company that manufactures toothpaste is studying five different package designs. Assuming that one design is just as likely to be selected by a consumer as any other design, what selection probability would you assign to each of the package designs? In an actual experiment, 100 consumers were asked to pick the design they preferred. The following data were obtained. Do the data confirm the belief that one design is just as likely to be selected as another? Explain.

Design	Number of Times Preferred
1	5
2	15
3	30
4	40
5	10

(10) A CBS News/New York Times poll of 1,000 adults in the United States asked the question, "Do you think global warming will have an impact on you during your lifetime?" (CBS News website). Consider the responses by age groups shown below.

	Age	
Response	18–29	30+
Yes	134	293
No	131	432
Unsure	2	8

a. What is the probability that a respondent $18-29$ years of age thinks that global warming will not have an impact during his/her lifetime?

b. What is the probability that a respondent 30+ years of age thinks that global warming will not have an impact during his/her lifetime?

c. For a randomly selected respondent, what is the probability that a respondent answers yes?

d. Based on the survey results, does there appear to be a difference between ages $18-29$ and 30+ regarding concern over global warming?

(11) A survey of magazine subscribers showed that 45.8% rented a car during the past 12 months for business reasons, 54% rented a car during the past 12 months for personal reasons, and 30% rented a car during the past 12 months for both business and personal reasons.

a. What is the probability that a subscriber rented a car during the past 12 months for business or personal reasons?

b. What is the probability that a subscriber did not rent a car during the past 12 months for either business or personal reasons?

(12) Suppose that a particular NBA player makes 93% of his free throws. Assume that late in a basketball game, this player is fouled and is awarded two free throws.

a. What is the probability that he will make both free throws?

b. What is the probability that he will make at least one free throw?

c. What is the probability that he will miss both free throws?

d. Late in a basketball game, a team often intentionally fouls an opposing player in order to stop the game clock. The usual strategy is to intentionally foul the other team's worst free-throw shooter. Assume that the team's worst free throw shooter makes 58% of his free-throw shots. Calculate the probabilities for this player as shown in parts (a), (b), and (c), and show that intentionally fouling this player who makes 58% of his free throws is a better strategy than intentionally fouling the player who makes 93% of his free throws. Assume as in parts (a), (b), and (c) that two free throws will be awarded.

(13) According to a 2018 article in Esquire magazine, approximately 70% of males over age 70 will develop cancerous cells in their prostate. Prostate cancer is second only to skin cancer as the most common form of cancer for males in the United States. One of the most common tests for the detection of prostate cancer is the prostate-specific antigen (PSA) test. However, this test is known to have a high false-positive rate (tests that come back positive for cancer when no cancer is present). Suppose there is a .02 probability that a male patient has prostate cancer before testing. The probability of a false-positive test is .75, and the probability of a false-negative (no indication of cancer when cancer is actually present) is .20.

a. What is the probability that the male patient has prostate cancer if the PSA test comes back positive?

b. What is the probability that the male patient has prostate cancer if the PSA test comes back negative?

c. For older men, the prior probability of having cancer increases. Suppose that the prior probability of the male patient is .3 rather than .02. What is the probability that the male patient has prostate cancer if the PSA test comes back positive? What is the probability that the male patient has prostate cancer if the PSA test comes back negative?

d. What can you infer about the PSA test from the results of parts (a), (b), and (c)?

(14) A study by Forbes indicated that the five most common words appearing in spam e-mails are *shipping!, today!, here!, available, and fingertips!* Many spam filters separate spam from ham (e-mail not considered to be spam) through ap- plication of Bayes' theorem. Suppose that for one e-mail account, 1 in every 10 messages is spam and the proportions of spam messages that have the five most common words in spam e-mail are given below.

shipping! .051
today! .045
here! .034
available .014
fingertips! .014

Also suppose that the proportions of ham messages that have these words are

shipping! .0015
today! .0022
here! .0022
available .0041
fingertips! .0011

a. If a message includes the word *shipping!*, what is the probability the message is spam? If a message includes the word *shipping!*, what is the probability the message is ham? Should messages that include the word *shipping!* be flagged as spam?

b. If a message includes the word *today!*, what is the probability the message is spam? If a message includes the word *here!*, what is the probability the message is spam? Which of these two words is a stronger indicator that a message is spam? Why?

c. If a message includes the word *available*, what is the probability the message is spam? If a message includes the word *fingertips!*, what is the probability the message is spam? Which of these two words is a stronger indicator that a message is spam? Why?

d. What insights do the results of parts (b) and (c) yield about what enables a spam filter that uses Bayes' theorem to work effectively?