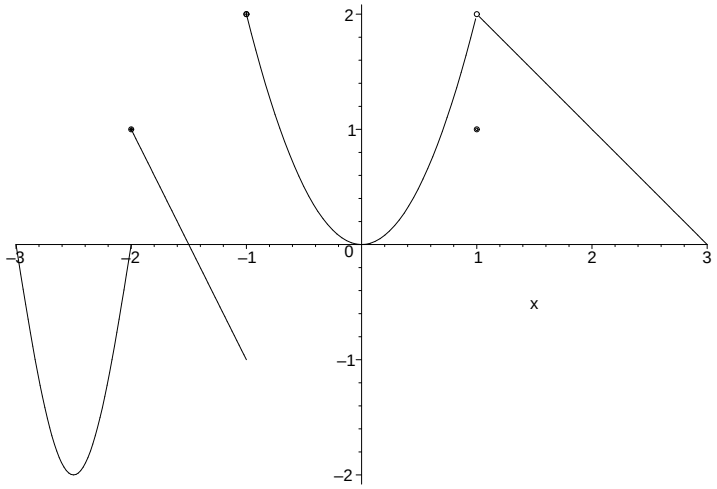
Mathematics I **Worksheet 4** Name:

Use the graph below to answer the following questions:




1. List any points at which $f(x)$ is not continuous.

2. Are any of the discontinuities removable? If so, how could you define the function to make it continuous there? Are any of the discontinuties jump discontinuities?

3. Find $a$ and $b$ so that $f(x) = \begin{cases} \frac{\sin(2x)}{x}, & x < 0 \\ a, & x = 0 \\ be^x + 3, & x > 0 \end{cases}$ is continuous.

4. Use the log laws to simplify $\log_3\left(\frac{x^2y^{3/4}}{z^2+1}\right)$.

5. Find $\log_2\left(\frac{1}{4}\right)$ without using your calculator.

6. Find $\log_3\left(\frac{1}{9}\right) + \log_3(27)$ without using your calculator.

7. Solve $\ln(x+3) + \ln(x-1) = \ln(12)$.

8. Expand $\ln\left(\frac{y^3(x+3)^2}{x^4+5}\right)$.

9. Find $\lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$.

10. Use the Squeeze Theorem to show that

    (a) $\lim_{x\to 0} \sqrt{x^3 + x^2} \sin \dfrac{\pi}{x} = 0.$

    (b) $\lim_{x\to 0} x^4 \cos \dfrac{2}{x} = 0.$

11. Explain why the function is discontinuous at the given number $a$. Sketch the graph of the function near $a$.

    (a) $f(x) = \dfrac{1}{x+2},\ a = -2$

    (b) $f(x) = \begin{cases} \dfrac{1}{x+2} & \text{if } x \neq -2, a = -2 \\ 1 & \text{if } x = -2 \end{cases},$

    (c) $f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1, a = 1 \\ \dfrac{1}{x} & \text{if } x \geq 1 \end{cases},$

    (d) $f(x) = \begin{cases} cosx & \text{if } x < 0, a = 0 \\ 0 & \text{if } x = 1 \\ 1 - x^2 & \text{if } x > 0 \end{cases},$

12. Use IVT to show that there is a root of given equation in the specified interval.

    (a) $x^4 + x - 3 = 0, (1, 2)$

    (b) $sinx = x^2 - x, (1, 2)$

13. Which of the following functions has a removable discontinuity?

    (a) $f(x) = \dfrac{x^4 - 1}{x - 1}, a = 1$

    (b) $f(x) = \dfrac{x^3 - x^2 - 2x}{x - 2}, a = 2$