Differential Equations **Worksheet 1** Name:

1. State the order and the degree of the given ordinary differential equation. Determine whether the equation is linear or nonlinear.
   (a) $t^5 y^{(4)} - t^3 y'' + 6y = 0$

   (b) $\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

   (c) $\dfrac{d^2u}{dr^2} + \dfrac{du}{dr} + u = \cos(r+u)$

   (d) $(1-x)y'' - 4xy' + 5y = \cos(x)$

   (e) $x\dfrac{d^3y}{dx^3} - \left(\dfrac{dy}{dx}\right)^4 = 0$

   (f) $\dfrac{dy}{dx} = \dfrac{\cos y}{y}$

2. Verify that the indicated expression is an implicit solution of the given first-order differential equation.
$$2xydx + (x^2 - y)dy = 0; \quad -2x^2y + y^2 = 1$$

3. What function do you know from calculus is such that its second derivative is itself? Write the answer in the form of a second-order differential equation with a solution.

4. A one parameter family (with paramater c) of solutions to the problem $y' = y - y^2$ is $y = \dfrac{1}{1 + ce^{-x}}$.
   (a) Find c so that $y(1) = -2$.

(b) Find c so that $y(-2) = -1$.

5. The general solution of the equation $y'' + 9y = 0$ is $y = c_1 \cos(3x) + c_2 \sin(3x)$. Find values of c_1 and c_2 so that $y(0) = -4$ and $y'(0) = -9$.

6. The general solution of the equation $y'' - 9y = 0$ is $y = c_1 e^{3x} + c_2 e^{-3x}$. Find values of c_1 and c_2 so that $y(0) = -7$ and $y'(0) = -3$.

7. Find corresponding differential equation for each curve families below.

   (a) $y = c_1 e^{-2x} + c_2 e^{3x}$.

   (b) $y = c_1 \cos x + c_2 \sin x$.

   (c) $y = c_1 x^2 + c_2$.

8. Solve the given differential equations by separation or variables.

   (a) $yy' = x(1 + y^2)$.

   (b) $y' = \dfrac{(1-y)^5}{4}$.

   (c) $y' = 5(1 + x)^4$.

   (d) $y' = y(y - 1)$.

   (e) $y' = 2xe^{x^2 - y}$.

9. Solve the separable initial value problem.

$$y' = 2x \cos(x^2)(1 + y^2), \quad y(0) = 1.$$