# Algebra Facts

***Special Products***

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$$

$$a^2 - 2ab + b^2 = (a-b)^2 \qquad a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$$

***Factoring***

$$a^2 - b^2 = (a+b)(a-b) \qquad a^3 + b^3 = (a+b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

***Exponent Properties***

$$a^n = a \cdot a \cdot \ldots \cdot a \quad \text{(n factors of a)}$$

$$a^0 = 1, \quad a \neq 0 \qquad a^{-n} = \frac{1}{a^n}, \quad a \neq 0 \qquad a^r a^s = a^{r+s} \qquad \frac{a^r}{a^s} = a^{r-s}, \quad a \neq 0$$

$$(a^r)^s = a^{rs} \qquad (ab)^r = a^r b^r \qquad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}, \quad b \neq 0 \qquad \left(\frac{a}{b}\right)^{-r} = \frac{b^r}{a^r}, \quad a, b \neq 0$$

***Radical Properties***

$$a^{1/n} = \sqrt[n]{a} \qquad a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m} \qquad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[b]{n}}$$

***Quadratic Formula***

If $ax^2 + bx + c = 0$, for $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

***Cartesian Coordinates***

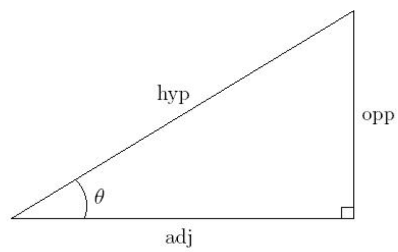
Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$. Then

1. The distance $d$ between $P_1$ and $P_2$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. The slope $m$ of the line segment $P_1P_2$ is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
3. The slope intercept form of a line is $y = mx + b$.
4. The point slope form of a line is $y - y_1 = m(x - x_1)$.

***Logarithm Properties***

1. $\log_b MN = \log_b M + \log_b N$
2. $\log_b \dfrac{M}{N} = \log_b M - \log_b N$
3. $\log_b M^k = k \log_b M$

# Trigonometry Facts



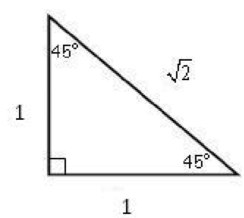

Definition Let $\theta$ be an acute($0 < \theta < \frac{\pi}{2}$) angle of a right triangle. The six trigonometric functions of the angle $\theta$ are defined as follows:

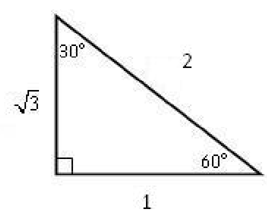
$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Since the angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{2}$ appear frequently in trigonometry (and calculus), it is strongly recommended that your learn to construct the following special right triangles.




***Sines, Cosines, and Tangents of Special Angles***

$$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan\frac{\pi}{4} = 1$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

***Trigonometric Identities***

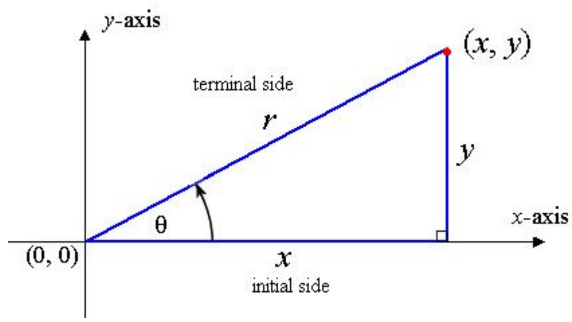
In trigonometry and calculus, a great deal of time is spent studying the relationship between trigonometric functions. Of particular importance are the following Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Proof

$$1 + \tan^2\theta = 1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

Previously, the definitions of the trigonometric functions were restricted to *acute* angles. Now we extend the definition to cover *any* angle.




Definition Let $\theta$ be an angle in standard position with $(x, y)$ a point on the terminal side of $\theta$ and $r = \sqrt{x^2 + y^2}$ in the figure above. The six trigonometric functions of the angle $\theta$ are defined as follows:

$$\sin\theta = \frac{y}{r}, \quad r \neq 0 \qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$$

$$\cos\theta = \frac{x}{r}, \quad r \neq 0 \qquad \sec\theta = \frac{r}{x}, \quad x \neq 0$$

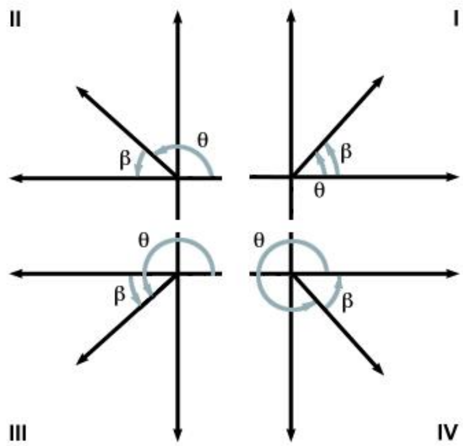
$$\tan\theta = \frac{y}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

***Reference Angles***

The values of the trigonometric functions of angles greater than $\frac{\pi}{2}$ can be determined from their values at corresponding acute angles called **reference angles**.

<u>*Definition*</u> Let $\theta$ be an angle in standard position. Its **reference angle** is the acute angle $\beta$ formed by the terminal side of $\theta$ and the horizontal axis.

The figure below shows the reference angle $\beta$ for $\theta$ in all four quadrants.




Below is a chart that will help in the easy calculation of reference angles. For angles in the first quadrant, the reference angle $\beta$ is equal to the given angle $\theta$. For angles in other quadrants, reference angles are calculated this way:

| **quadrant** | **reference angle** $\beta$ |
|---|---|
| I | $\beta = \theta$ |
| II | $\beta = \pi - \theta$ |
| III | $\beta = \theta - \pi$ |
| IV | $\beta = 2\pi - \theta$ |